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DETERMINATION OF THE THERMAL DIFFUSIVITY OF VERMICOMPOST AND THE EFFECT OF MOISTURE AND TEMPERATURE ON IT USING THE 1D FOURIER CYLINDRICAL SOLUTION


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Abstract. Earthworm is the crucial actor in vermicompost production. Separating the worms with the least harm will decrease the costs of the vermicompost production. One of the key factors in using heat for separation of the worms is the determination of the thermal diffusivity of vermicompost. The thermal diffusivity of vermicompost has been determined by using a one-dimensional (1D) Fourier equation applied to a cylinder. The measurements have been carried out at 40, 50, 60, 70 °C temperature levels and moisture contents of 10, 20, 30, 40, 50 % (wet base). The experiments have been conducted by heating vermicompost in an insulated box. The thermal diffusivity is estimated by keeping time–temperature records. The values of the thermal diffusivity of vermicompost ranged from 1.3973×10⁻⁷ to 2.861×10⁻⁷ m² s⁻¹. By increasing the moisture content of vermicompost from 10% to 50%, the thermal diffusivity of vermicompost is decreased. The thermal diffusivity of vermicompost increases with increases in the temperature from 40 °C to 70 °C. The relationship between thermal diffusivity (α), moisture content (MC), and temperature (T) is described using a polynomial model, with the coefficient of determination being 0.99.

Keywords: Fourier’s equation, moisture content, polynomial model, temperature, cooling time

Introduction

One of the most important methods for processing municipal wastes is recycling them to vermicompost. Because of this process, in addition to reducing health and environmental problems, significant quantities of organic fertilizers, known as vermicompost, are produced. Vermicompost is a biological and organic fertilizer, which is produced by means of constant and calm passing of decaying organic matter from the digestive tract of earthworms and the excretion of these substances from the worm body (Ismail et al., 2003). These materials will be smeared with mucus, vitamins, and enzymes while passing through the body of the worm and will be ultimately enriched as an organic fertilizer. This type of fertilizer can be produced and used for building and improving soil nutrients. Therefore, vermicompost is a worm extract with a percentage of organic matter and food and carbohydrate substrates (Sathe, 2005). Farmers use this type of compost, which is a mixture of processed organic matter, due to its unique and
specific modification effects on the physical, chemical, and biological properties of soil and consequently growth and increase of the product (Manna et al., 2003). Effective separation of earthworms from the organic media after completing a vermicomposting batch process could increase the rate of waste bioprocessing by earthworms and minimize the monetary investment in earthworms for such operations.

There are various methods for separating worms from vermicompost. Among them are mechanical separation, reducing the moisture of the vermicompost, using infra-red rays and using electric shock (Chaoui and Keener, 2008). The mechanical separation is of little efficiency and kills a large number of worms. Moisture Reduction is time-consuming and needs large space (Fieldson, 1988). Using infra-red rays and electric shock kills too many worms (Chaoui and Keener, 2008).

In the controlled-heat and layered method of separation the worms are made to move to layers with lower temperature and with their arrival on the surface they can be separated from the vermicompost. The thermal properties of vermicompost are needed to predict heat transfer and increasing temperature in vermicompost for separating earthworms by heating from the organic media after completing vermicomposting.

The physical significance of thermal diffusivity is associated with the propagation of heat into the medium during temperature changes with time. The higher the thermal diffusivity, the faster the propagation of heat into the medium (Jain and Pathare, 2007). In any constant temperature of the thermal source, the more the value of the thermal diffusivity coefficient, the faster the object gets into thermal stability. Thus, there will be fewer temperature differences in the object (Bairi et al., 2007).

The measurement of thermal diffusivity can be divided into two groups: direct measurement and measurement by the values of thermal conductivity, namely specific heat and density, in an experimental manner (indirect prediction). The best method is the direct measurement of thermal diffusivity (Dickerson, 1965). There are four common methods to determine the thermal diffusivity experimentally: least squares estimation, use of heat penetration data, use of time–temperature charts, and use of analytical solutions (Singh, 1982). The thermal diffusivity of cheddar cheese was determined by using the analytical solution for an infinite cylinder with experimentally obtained time–temperature data (Marschoun et al., 2001). The thermal diffusivity of pet food was measured by center temperature data (Kee et al., 2002). The thermal diffusivity of Marquis wheat seed was studied in bulk at 9.2% (dry base) of the moisture content, and the value of thermal diffusivity was determined by indirect method (Babbit, 1945). The thermal diffusivity of rough rice was obtained in the moisture range of 12% to 20% (d.b) (Wratten et al., 1969). The thermal diffusivity of dairy cattle manure was measured in different moisture and temperature range (Amin-nayyeri et al., 2009). The thermal diffusivity of the sawdust was determined by using heat conduction and specific heat capacity in an indirect way (Ahn et al., 2009).

The objectives of this study were to determine the thermal diffusivity of vermicompost and to develop a mathematical model of the thermal diffusivity of vermicompost within the experimented temperature and moisture content limits. The research results have potential applications in separating earthworms by heating from the organic media after completion of vermicomposting.
Material and Methods

Sample preparation

The vermicompost used in this study produced at the Vermicompost production unit located at the Vermicompost Production Industrial Center in Tehran, Iran.

The particle size of the tested vermicompost was 38.9% with a size of 0.3, 44.85% with a size of 0.6 and 16.25% with a size of 1.18 mm and was determined as powder particles.

To determine the initial moisture content of the vermicompost, 100 g of vermicompost according to ASAE S358.2 standard was placed in the oven at 103 ± 3 °C for 48 hours. (ASAE Standards S269.4, 1998).

Experimental setup

The vermicompost sample was put in an aluminum cylinder with a length of 150 mm and diameter of 7.5 mm. The very thin wall of the cylinder had very low thermal resistance to conduction. A K-type thermocouple was fixed on the central axis and at the middle length of the sample-holder cylinder to sense the central temperature of the sample.

The cylinder was put into water with a certain temperature until the sample temperature was changed by the equilibration on the water. Next, the sample-holder cylinder was cooled by a water bath at temperatures of 30, 40, 50, 60 °C for the initial temperature of the sample (T0) 50, 60, 70, 80 °C, respectively. The temperature of the sample center was measured and recorded in a data logger (CHY 502 A, Taiwan) at an interval time of 2 min. Figure 1 shows the schematic of the apparatus used for measuring the thermal diffusivity of the vermicompost sample.

Figure 1. Schematic of the apparatus used for measuring the thermal diffusivity of vermicompost
Theoretical principle

Thermal diffusivity includes the effects of properties like mass density ($\rho$), thermal conductivity ($k$), and specific heat capacity ($C_p$) this is described by Equation 1 (Mohsenin, 1980).

$$\alpha = \frac{k}{\rho C_p} \quad \text{(Eq. 1)}$$

When heat is transferred by conduction through solid material it generates a temperature field can be described by Fourier equation in cylindrical coordinates:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{(Eq. 2)}$$

Where: $T$ is temperature in °C; $r$ is radius in m; $x$ is axial coordinate in m; a thermal diffusivity in m$^2$ s$^{-1}$; and $t$ is time in s.

Considering the very small internal temperature gradient, the derivative $\frac{\partial^2 T}{\partial x^2}$ is ignored. As the cylinder is long enough to consider that the heat exchanges are made quasi-exclusively through the lateral surface, the problem is simplified and becomes one-dimensional (1D). Thus, the Equation 2 can be rewritten as Equation 3:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{(Eq. 3)}$$

Equation 3 can be solved as Equation 4 (Bairi and Laraqi, 2003):

$$\theta = \frac{T_m - T_e}{T_0 - T_e} = 2 \sum_{n=1}^{\infty} \exp(-\varphi_n^2 r^2) \frac{1}{\varphi_n} \frac{J_1(\varphi_n)}{J_0(\varphi_n) + J_1(\varphi_n)} J_0(\varphi_n r^*) \quad \text{(Eq. 4)}$$

Equation 4 can become linear after a long time and then only the first term of the series solution can be acceptable to solve it. Thus, Equation 4 can be rewritten as Equation 5:

$$\theta = \frac{T_m - T_e}{T_0 - T_e} = \beta \exp(-\varphi_1^2 r^2) \quad \text{(Eq. 5)}$$

Where the constant $\beta$ is given by Equation 6:

$$\beta = 2 \frac{1}{\varphi_n} \frac{J_1(\varphi_n)}{J_0^2(\varphi_1) + J_1^2(\varphi_1)} J_0(\varphi_n r^*) \quad \text{(Eq. 6)}$$

The Fourier number is given by Equation 7:
Therefore, Equation 5 can be rewritten as Equation 8:

\[ \theta = \frac{T_m - T_e}{T_0 - T_e} = \beta \exp \left(-\phi_1 \frac{\alpha t}{r^2}\right) \]  

(Eq. 8)

The parameters are defined below:

\( \theta \): dimensionless temperature ratio, \( T_0 \): initial temperature of the vermicompost in °C, \( T_m \): temperature of the center of vermicompost in °C, \( T_e \): temperature of the environment in °C, \( F_{o_c} \): Fourier number in dimensionless, \( r \): radial cylindrical coordinate, \( \phi_n, \phi_1 \): positive roots of the Bessel’s equation, \( J_0 \) and \( J_1 \): the Bessel function of first kind of order 0 and 1, \( r^* \): the reduced radius (\( r^* = r/R \)), and \( t \): time in s.

The value for the first root \( \phi_1 \) is 2.405. Thus, thermal diffusivity can be calculated by measuring the slope of Equation 9 (Bairi et al., 2007):

\[ \alpha = \frac{1}{\tau} \left( \frac{R^2}{2.405} \right)^2 \]  

(Eq. 9)

Where \( \tau \) is the time constant.

**Results and Discussion**

The logarithmic dimensionless temperature (\( Ln\theta \)) was plotted against the cooling time for the vermicompost sample with four variable temperatures with the moisture content of 40% (Fig. 2). Figure 2 represents the linear relationship with a negative slope between the logarithmic dimensionless temperature and the cooling time for four temperature levels. Moreover, the values of slope (\( |A| \)) increased with decreases in the temperature.

![Figure 2. Evaluation of dimensionless temperature (Ln\theta) during the cooling time for vermicompost at 40% and different temperatures](image-url)
A regression analysis was performed to show the linear relationship between the cooling time and the logarithmic dimensionless temperature. Table 1 illustrates the parameters of the linear relationship and correlation coefficients between logarithmic dimensionless temperature and cooling time. The coefficient of determination, $R^2$, ranged from 0.9924 to 0.9996, and the standard error was found to vary from 0.01821 to 0.09386. The higher $R^2$ and lower standard error represent the higher accuracy of the computation of thermal diffusivity. According to Table 1, the value of slope ($A$) increases with increases in temperature, while it decreases with increases in the moisture content from 10% to 50% on wet base (w.b).

Table 1. Relationship between the dimensionless temperature $\ln(\theta)$ and the cooling time $t$ for vermicompost [$\ln (\theta) =At+B$]

<table>
<thead>
<tr>
<th>Moisture content %</th>
<th>Temperature (°C)</th>
<th>Coefficient A</th>
<th>Coefficient B</th>
<th>$R^2$</th>
<th>es</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>40</td>
<td>-0.01720</td>
<td>-0.0675</td>
<td>0.9949</td>
<td>0.04452</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>-0.02124</td>
<td>0.0503</td>
<td>0.9973</td>
<td>0.07146</td>
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<tr>
<td></td>
<td>60</td>
<td>-0.02428</td>
<td>-0.02731</td>
<td>0.9933</td>
<td>0.05863</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>-0.02942</td>
<td>-0.2581</td>
<td>0.9924</td>
<td>0.08509</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
<td>-0.01637</td>
<td>-0.0885</td>
<td>0.9946</td>
<td>0.05698</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>-0.02022</td>
<td>-0.04070</td>
<td>0.9994</td>
<td>0.02730</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>-0.02326</td>
<td>-0.0242</td>
<td>0.9992</td>
<td>0.03646</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>-0.02824</td>
<td>-0.024</td>
<td>0.9981</td>
<td>0.05575</td>
</tr>
<tr>
<td>30</td>
<td>40</td>
<td>-0.01564</td>
<td>-0.1373</td>
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<tr>
<td></td>
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<td>0.02858</td>
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<td>0.07743</td>
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<tr>
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<td>-0.2677</td>
<td>0.9970</td>
<td>0.06243</td>
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<td>-0.02621</td>
<td>0.3522</td>
<td>0.9957</td>
<td>0.08282</td>
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<tr>
<td>40</td>
<td>40</td>
<td>-0.01301</td>
<td>-0.0640</td>
<td>0.9992</td>
<td>0.01947</td>
</tr>
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<td>-0.01790</td>
<td>0.09460</td>
<td>0.9936</td>
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</tr>
<tr>
<td></td>
<td>60</td>
<td>-0.02144</td>
<td>-0.05820</td>
<td>0.9996</td>
<td>0.01821</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>-0.02533</td>
<td>0.01810</td>
<td>0.9938</td>
<td>0.09386</td>
</tr>
<tr>
<td>50</td>
<td>40</td>
<td>-0.01437</td>
<td>0.06290</td>
<td>0.9994</td>
<td>0.01947</td>
</tr>
<tr>
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<td>50</td>
<td>-0.01731</td>
<td>0.06460</td>
<td>0.9936</td>
<td>0.05205</td>
</tr>
<tr>
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<td>-0.02070</td>
<td>-0.05666</td>
<td>0.9996</td>
<td>0.01821</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>-0.02366</td>
<td>0.06410</td>
<td>0.9938</td>
<td>0.09386</td>
</tr>
</tbody>
</table>

The thermal diffusivity was calculated by using Equation 9. The value of thermal diffusivities, $\alpha$, in m$^2$s$^{-1}$ ranged from $1.3973 \times 10^{-7}$ to $2.861 \times 10^{-7}$ m$^2$s$^{-1}$. These values are within the general range of the thermal diffusivity of agricultural materials. The thermal diffusivity of Marquis wheat seed was studied in bulk at 9.2% (d.b) of the moisture content, and the value of thermal diffusivity was equal to $1.15 \times 10^{-7}$ m$^2$s$^{-1}$ (Babbit, 1945). The thermal diffusivity of rough rice was obtained from $0.856 \times 10^{-7}$ to $1.05 \times 10^{-7}$ m$^2$s$^{-1}$ in the moisture range of 12% to 20% (d.b) (Wratten et al., 1969). The value of thermal diffusivity of manure was reported to be $1.65 \times 10^{-7}$ m$^2$s$^{-1}$ (Houkom et al., 1974). The thermal diffusivity of dairy cattle manure was measured from $0.904 \times 10^{-7}$ to $2.11 \times 10^{-7}$ m$^2$s$^{-1}$ (Amin-nayyeri et al., 2009). Table 2 presents the variance analysis results of the thermal diffusivity of the vermicompost sample that include the effects of temperature and moisture parameters as well as of their interactions. As presented in Table 2, the temperature and moisture parameters have become significant (P <0.001), but their interaction happens to be non-significant.
Table 2. The variance analysis results of the thermal diffusivity of vermicompost

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>Degree of freedom</th>
<th>Mean sum of squares</th>
<th>F-value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>5.69E-15</td>
<td>19</td>
<td>2.99E-16</td>
<td>2133.05**</td>
<td>0.0000</td>
</tr>
<tr>
<td>T</td>
<td>5.52E-15</td>
<td>3</td>
<td>1.84E-15</td>
<td>1311.03***</td>
<td>0.0000</td>
</tr>
<tr>
<td>MC</td>
<td>1.67E-16</td>
<td>4</td>
<td>4.20E-17</td>
<td>298.75***</td>
<td>0.0000</td>
</tr>
<tr>
<td>MC×T</td>
<td>1.22E-21</td>
<td>12</td>
<td>1.02E-22</td>
<td>0.0007ns</td>
<td>0.0000</td>
</tr>
<tr>
<td>Residual</td>
<td>5.62E-18</td>
<td>40</td>
<td>1.41E-19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5.70E-15</td>
<td>78</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*** Highly significant; ns non-significant

Figure 3 presents the effect of moisture content on the thermal diffusivity of vermicompost. By increasing the moisture content of vermicompost from 10% to 50% (w.b), thermal diffusivity decreases from $2.861\times10^{-7}$ to $1.3973\times10^{-7}$ m$^2$ s$^{-1}$. The results for thermal diffusivity of vermicompost are in general agreement with the findings of some previous researchers. The thermal diffusivity of gua seed decreased from $9.311\times10^{-8}$ m$^2$ s$^{-1}$ to $8.5\times10^{-8}$ m$^2$ s$^{-1}$ as the moisture content increased (Avira et al., 2008). The investigation showed that the thermal diffusivity of millet grains decreased from $2.03\times10^{-7}$ to $1.52\times10^{-7}$ m$^2$ s$^{-1}$. (Subramanian and Viswanathan, 2003).

Figure 3. Thermal diffusivity of vermicompost versus moisture content at different temperatures

Figure 4 shows the effect of temperature on the thermal diffusivity of vermicompost. The thermal diffusivity of vermicompost increased with rising temperatures (Fig. 4). Moreover, the line of 10% moisture content is higher than other moisture content levels. The thermal diffusivity of solid food increased almost linearly with increases in the temperature (Fito et al., 1984). The investigation showed that the thermal diffusivity of cumin seed increased from $0.653\times10^{-7}$ to $16.64\times10^{-7}$ m$^2$ s$^{-1}$ (Singh and Goswami, 2000).
A polynomial relationship among the thermal diffusivity ($\alpha$), temperature ($T$), and moisture content ($MC$) was obtained by multiple regression analysis with a value of 0.99 for the coefficient of determination as Equation 10:

$$\alpha = 1.01 \times 10^{-7} + 1.35 \times 10^{-9} T + 1.93 \times 10^{-11} T^2 - 1.3 \times 10^{-9} MC + 4.57 \times 10^{-12} MC^2 \quad \text{(Eq. 10)}$$

Table 3 shows that the effect of temperature (high F-value) on the thermal diffusivity is more than that of moisture content. The significance of respective regression coefficients in Equation 10 also confirms the results.

Table 3. Analysis of variance (ANOVA) for effect of parameters in Equation 10 on thermal diffusivity of vermicompost

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>Degree of freedom</th>
<th>Mean sum of squares</th>
<th>Fcal-value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>3.47E-14</td>
<td>4</td>
<td>8.67E-15</td>
<td>38403.03***</td>
<td>0.0000</td>
</tr>
<tr>
<td>T</td>
<td>2.55E-16</td>
<td>1</td>
<td>2.55E-16</td>
<td>1130.22***</td>
<td>0.0000</td>
</tr>
<tr>
<td>$T^2$</td>
<td>7.48E-17</td>
<td>1</td>
<td>7.48E-17</td>
<td>331.05***</td>
<td>0.0000</td>
</tr>
<tr>
<td>MC</td>
<td>3.02E-17</td>
<td>1</td>
<td>3.02E-17</td>
<td>134.02***</td>
<td>0.0000</td>
</tr>
<tr>
<td>$MC^2$</td>
<td>3.43E-17</td>
<td>1</td>
<td>3.43E-17</td>
<td>151.97***</td>
<td>0.0000</td>
</tr>
<tr>
<td>Residual</td>
<td>3.38E-18</td>
<td>15</td>
<td>2.25E-19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3.47E-14</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*** Highly significant

Any comparison between the measured and estimated values of the thermal diffusivity (Fig. 5) showed that the maximum differences were within $\pm 0.01 \times 10^{-7} \text{ m}^2\text{s}^{-1}$, showing reasonable accuracy for estimating the thermal diffusivity.
Conclusions

The thermal diffusivity of vermicompost was determined using a 1D solution of the Fourier equation of heat. The thermal diffusivity of vermicompost is dependent on the latter’s moisture content and temperature. The effect of temperature on the thermal diffusivity was more than that of moisture content. By increasing the moisture content of the vermicompost sample from 10% to 50%, its thermal diffusivity decreased from $2.861 \times 10^{-7}$ to $1.3973 \times 10^{-7}$ m$^2$ s$^{-1}$. The thermal diffusivity of vermicompost increases with rising temperature. The developed model can also be used acceptably within the range of variables studied in this research where the thermal diffusivity of the vermicompost ranged from $1.3973 \times 10^{-7}$ to $2.861 \times 10^{-7}$ m$^2$ s$^{-1}$. The resulting thermal diffusivity of vermicompost can be used to develop heat transport models for the design of more optimal temperature control in composting systems.

REFERENCES


