MODELING STEM PROFILE OF CAUCASIAN FIR AND ORIENTAL SPRUCE MIXED STANDS IN TURKEY USING NONLINEAR MIXED-EFFECTS MODELS

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Abstract. The objective of this study is to develop segmented polynomial taper equations using the nonlinear mixed-effects modeling approach for Caucasian fir (*Abies nordmanniana* (Steven) Spach. subsp. *nordmanniana*) **-** Oriental spruce (*Picea orientalis* (L.) Link.) mixed stands in Artvin-Ardanuç region of Turkey. For this purpose, the data obtained from 213 felled sample trees in total, which are 107 fir and 106 spruce, were used. Jiang et al.'s (2005) stem profile model produced the best prediction results for both tree species. The R^2_{adj}, RMSE, $\bar{E}$, MAE, AIC and BIC values of the model were found as 98.7% and 98.3%, 1.700 and 1.814 cm, 0.143 and 0.167 cm, 1.179 and 1.269 cm, 4142.2 and 3951.7, 4148.5 and 3957.8 for fir and spruce, respectively. Based on AIC, BIC and -2LnL criteria, the model including random-effects in two parameters (b_3 and b_4) were the best for both tree species. This mixed-effects model with two random parameters showed homogeneous residual variance and autocorrelation was almost removed. Mixed-effects models provided much better fits than their fixed model counterparts for both species. In addition, among 12 different calibration choices, the best results were obtained with the choice involving the measurement of five diameters that divide the sample tree measurements into two equal parts for fir and of four diameters that bottom of the sample tree for spruce.
Keywords: *stem diameter, segmented polynomial taper models, calibration, random parameters, Artvin*

Introduction

The correct calculation of the volume of trees and stands, and of the distribution of this volume among different commercial classes, is of great importance when formulating forest management plans and making projections concerning the future of the forestry products sector. Among the methods for the accurate estimation of tree volumes, the most effective and versatile approach is the stem taper model (Jiang et al., 2005; Özçelik and Brooks, 2012). Using this method, detailed estimations can be made of the types of wood that can be obtained from trees in demand under the ever-changing market conditions (Ercanlı and Kahriman, 2013; Kumaş and Kahriman, 2015).

There are two important reasons for the continuous increase in the number of studies on stem taper: The first reason is that the stem forms of all trees are not alike and that a theory has yet to be developed that reflects these differences in form; and the second is that a single method is yet to be developed that considers the changing standards in wood types under the ever-changing market conditions (Newnham, 1992; Diéguez-Aranda et al., 2006; Özçelik and Yaşar, 2015). Thus, it is quite easy to determine both the total stem volume and the merchantable volume using a simple stem taper model.

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Stem profile equations not only identify the stem form but also estimate (i) the total stem volume, (ii) the merchantable stem volume, (iii) the volume of the stem section between any two heights, (iv) the volume of the stem section between any two diameters, (v) the stem diameter at any height, and (vi) the height at any stem diameter (Kozak, 2004; Özçelik et al., 2011).

Many different stem taper models have been developed over the last century, and they can be divided into three categories (Diéguez-Aranda et al., 2006) as follows: simple polynomial taper equations (Demaerschalk, 1972, 1973), variable-form taper equations (Newnham, 1992; Kozak, 2004) and segmented polynomial taper equations (Max and Burkhart, 1976; Parresol et al., 1987; Fang et al., 2000; Jiang et al., 2005). The performances of these stem profile functions vary depending on the tree species studied, the quality of the dataset, and the form and type of the selected model (Sakıcı et al., 2008; Özçelik et al., 2011; Şenyurt et al., 2017). Since the stem taper models that are compatible with the models used in this study divide the tree stem into parts and define each part individually, they can be used to estimate the entire stem profile in the most realistic way possible (Green and Reed, 1985). Another advantage of these stem taper models over other such models is that they can be easily converted into volume equations for the calculation of volumes (Fang et al., 2000).

The diameter data obtained from the various heights of a single tree can be used to develop a stem taper model, which indicates that successive measurement values are interrelated (Özçelik and Yaşar, 2015). In such hierarchical data structures, where each tree has a unique stem development, the problem of “autocorrelation”, also known as “serial correlation”, may be observed, which means that different items of data are dependent on each other (Leites and Robinson, 2004). Such serial correlations among the sample data may result in the confidence intervals in the parameter estimations concerning stem taper and stem volume equations being estimated with a systematic error, thus affecting the reliability of the model results in an adversely way (Searle et al., 1992; Kurt, 2014). In this regard, for hierarchical data structures in which the assumption of data independence cannot be met, and the problem of serial correlation exists in the data, the use of Nonlinear Mixed-Effects Models is recommended, as this enables the modeling of the variance-covariance matrix structure (Littell et al., 2005).

Regarding their ranges, incremental growth characteristics and economic value, the Caucasian Fir (*Abies nordmanniana* (Steven) Spach. subsp. *nordmanniana*) and the Oriental Spruce (*Picea orientalis* (L.) Link.) are among the leading forest tree species in Turkey. According to the 2015 forest inventory of the General Directorate of Forestry, the total areas covered by fir and spruce are 884,781 ha and 322,857 ha, respectively (General Directorate of Forestry, 2015).

According to the 2015 forest inventory of the General Directorate of Forestry, approximately 38% of forestry areas in Turkey (8,394,788 ha) are mixed forests, while forest inventory data from 2006 states that around 707 m^3 (55%) of the total tree wealth of Turkey is composed of mixed forests (General Directorate of Forestry, 2015), which suggests that mixed forests and mixed stands constitute a significant portion of Turkey’s forestry lands.

The objectives of this research were to develop a stem taper and stem volume model that facilitates detailed volume estimations of the mixed stands of Caucasian fir and Oriental spruce that are abundant in the area covered by the Artvin Regional Directorate - Ardanuç Forest District Directorate. We fitted and evaluated taper functions using mixed-effect modelling techniques.

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Material and methods

Study site

In the Artvin-Ardanuç region, where mixed stands of Caucasian fir (*Abies nordmanniana* (Steven) Spach. subsp. *nordmanniana*) and Oriental spruce (*Picea orientalis* (L.) Link.) can be found (40°54'54" - 41°15'31" N, 41°53'47"' - 42°22'52" E), the monthly average temperature varies between 2.8–21.0°C (annual average: 12.3°C), with the lowest temperature varies between +9.5 and -16.1 °C, and the highest temperature reaches 43°C. While annual average precipitation is 700 mm, annual average relative humidity varies between 60 and 70% (Turkish State Meteorological Service, 2016). The climate in Ardanuç is a mixture of continental and Mediterranean climates, where summers are hot and dry, and winters are partially warmer and less rainy than the normal continental climate.

There is around 75,535.4 ha of Caucasian fir, and 44,001.3 ha of Oriental spruce stands in the Artvin Regional Directorate, while 38,810.3 ha of this area is composed of mixed stands of Caucasian fir and Oriental spruce. The area of the mixed stands of Caucasian fir and Oriental spruce in the Ardanuç Forest District Directorate, as the study site, is 6,487.6 ha (General Directorate of Forestry, 2015). The ranges of the Caucasian fir and Oriental spruce mixed stands in Turkey, and the distribution of these two species in the study site are given in *Figure 1*.

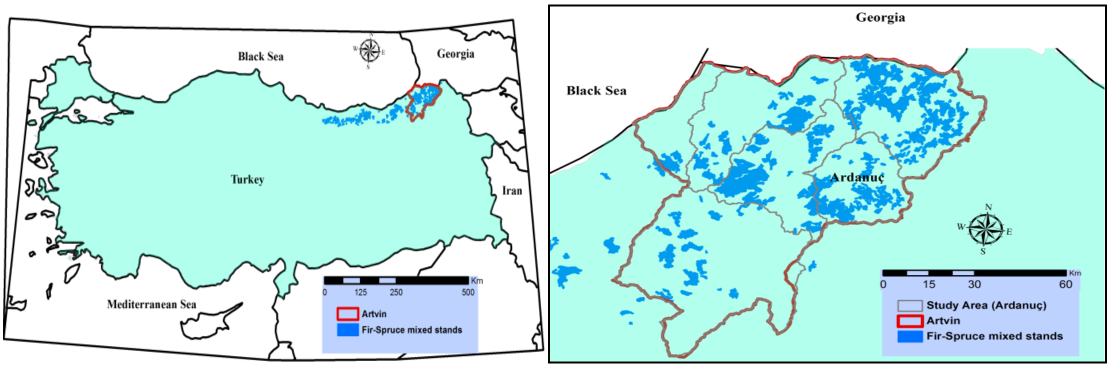

Figure 1. *Location of the study area*

Data

The data were obtained from 213 sample trees felled from the mixed stands of Caucasian fir and Oriental spruce within the boundaries of the Artvin Regional Directorate - Ardanuç Forest District Directorate, with 107 of them being fir and 106 being spruce. In selecting the sample trees, care was taken to ensure that the samples had a balanced distribution of various diameters and heights, and reflected the variation in volume development.

The sample trees used in this study were felled at the stump height level (0.3 m). Their stump diameter (0.30 m) and diameter at breast height (1.30 m) were measured, and after which, their diameter values were measured at 1 m intervals (2.3, 3.3, 4.3 m...). The total height values of the trees were measured with a steel measuring tape. If the tree stems were found to be oddly-shaped rather than cylindrical during measurement, two diameters perpendicular to the stem's cross section were measured, and their average was taken as the diameter measurement.

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The data used in this study were divided randomly into two groups, with one being used in the estimation of the parameters of stem taper models and the other being used in the determination of the calibration responses of such models. Approximately 80% of the data (85 fir and 84 spruce trees) were included in group I as model data, and the remaining 20% (22 fir and 22 spruce trees) were included in group II as control data. *Figures 2* and *3* show a scatter plot of relative diameter of these trees against their corresponding relative height of the data used, for both fitting and validation data sets.

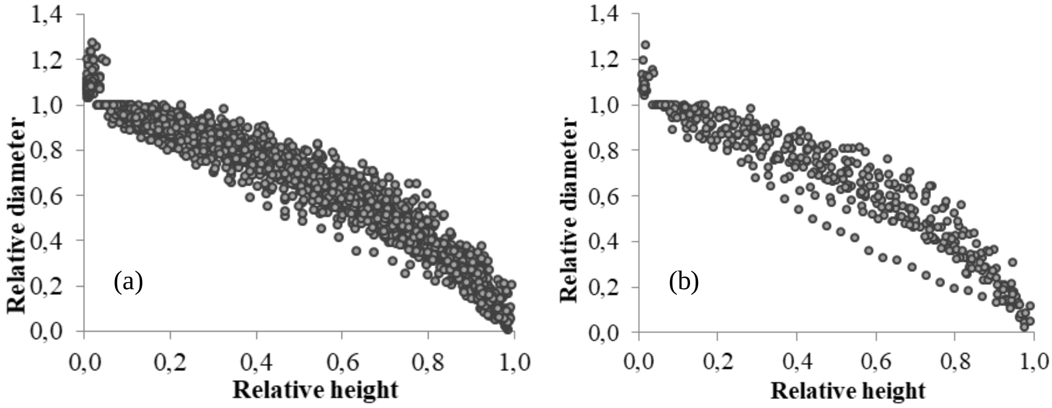

Figure 2. *Plot of the relative height versus relative diameter outside bark for (a) the fitting data points, and (b) the validation data points of Caucasian fir*

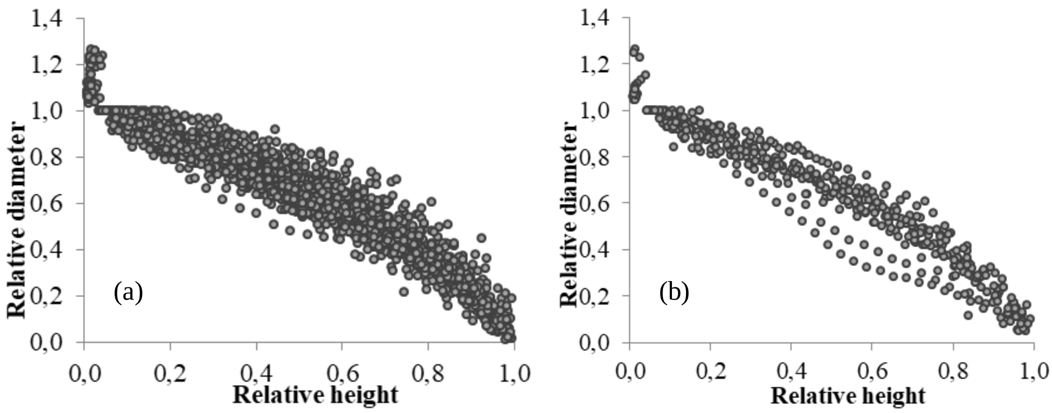

Figure 3. *Plot of the relative height versus relative diameter outside bark for (a) the fitting data points, and (b) the validation data points of Oriental spruce*

Table 1 provides the summary statistics of diameter at breast height, total height, tree volume, diameter outside bark at a height h and height above ground to the measurement point for model fitting and validation data set.

Selection of the best statistical regression models

The estimation of the parameters of these stem taper models was done with the NLIN procedure available in SAS/STAT® 9.3 software (SAS Institute, 2013). In this study, the adjusted coefficient of determination (R^2_{adj}), Root-Mean-Square Error (RMSE), Bias ($\bar{E}$), Mean Absolute Error (MAE), Akaike's Information Criterion (AIC) and

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Schwarz's Bayesian Information Criterion (BIC) were used to evaluate the performance of the stem taper models for both species. It is desired that R^2_{adj} be close to 1 and that the RMSE, $\bar{E}$, MAE, AIC, and BIC values be low (Castedo-Dorado et al., 2006). The model evaluation criteria were as follows (*Eqs. 1–6*):

$$R^2_{adj} = 1 - \frac{(n-1)\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{(n-p)\sum_{i=1}^{n}(y_i - \bar{y}_i)^2} \quad \text{(Eq.1)}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-p}} \quad \text{(Eq.2)}$$

$$\bar{E} = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)}{n} \quad \text{(Eq.3)}$$

$$MAE = \frac{\sum_{i=1}^{n}|y_i - \hat{y}_i|}{n} \quad \text{(Eq.4)}$$

$$AIC = -2\ln(L) + 2k \quad \text{(Eq.5)}$$

$$BIC = -2\ln(L) + k.\ln(n) \quad \text{(Eq.6)}$$

where n is the number of observations; p is the number of coefficient in the model; y_i, $\hat{y}_i$ and $\bar{y}$ are observed, predicted and average observed the diameters outside bark, respectively, k is the number of parameters in the model; L maximum likelihood (ML) value.

Table 1. *Descriptive statistics for Caucasian fir and Oriental spruce sample trees*

Species	Variable	Initial model development (80%)				Model validation (20%)			
		n	**Mean**	**S.D.**	**Range**	**n**	**Mean**	**S.D.**	**Range**
Caucasian fir	D (cm)	85	33.6	15.5	9.4-66.0	22	35.1	15.8	7.9-65.5
	H (m)	85	21.2	7.5	5.7-39.8	22	21.5	8.0	7.7-40.0
	v (m^3)	85	1.36	1.36	0.03-5.68	22	1.48	1.58	0.03-6.66
	d (cm)	1830	25.5	14.7	0.2-75.0	451	24.0	13.2	1.2-66.3
	h (m)	1830	12.0	8.2	0.3-39.3	451	11.2	7.5	0.3-34.3
Oriental spruce	D (cm)	84	32.6	14.9	8.0-80.0	22	36.6	16.4	8.7-72.1
	H (m)	84	20.2	6.7	6.8-38.5	22	21.3	6.5	7.5-31.1
	v (m^3)	84	1.14	1.24	0.02-6.21	22	1.42	1.22	0.02-4.13
	d (cm)	1703	23.3	13.8	0.4-86.0	466	25.6	14.6	1.0-76.4
	h (m)	1703	11.1	7.5	0.3-38.3	466	11.4	7.4	0.3-30.3

D = diameter at breast height; H = total height; d = diameter outside bark at height h; h = height above ground level, v = tree volume (m^3); n = number of observations; S.D. = standard deviation

The data were randomly divided into two groups as follows: (i) the data used for estimation of the parameters of the mixed-effects stem taper models; and (ii) the data

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used for the calibration of such models. Group I contains approximately 80% of all data (1830 stem diameter measurements of 85 sample fir and 1703 stem diameter measurements of 84 sample spruces), whereas group II contains approximately 20% of all data (451 stem diameter measurements of 22 sample fir and 466 stem diameter measurements of 22 sample spruce).

After identifying the best among the six stem taper models used in this study, the next step was the application of the Mixed-Effects Modeling process on the best stem taper model.

Statistical nonlinear mixed-effect modeling approach

In developing the stem taper models, the diameter values measured along the stem of each tree are used. When these diameter values, which are interdependent (serial correlation “autocorrelation”) are used in model development, the principle of data independence, as one of the fundamental assumptions of a regression analysis, is violated (Valentine and Gregorie, 2001), which may result in the confidence intervals of the parameters being estimated with a systematic error, the reliability of model results being adversely affected and erroneous estimations (Ye, 2005). To account for such serial correlations, the mixed-effects approach has been widely recommended in modelling, as this enables the modeling of the variance-covariance matrix structure (Littell et al., 2005).

Unlike in linear and nonlinear regression analyses, the model structure in a mixed-effects modeling approach is based on a structure in which the parameters are divided into two groups, namely parameters for fixed effects and those for random effects. While fixed-effect parameters refer to the general relationships that apply to the entire population, random-effect parameters refer to the variability among the different sampling units (sample trees). The equation structures of the mixed models are given below (*Eq. 7*):

$$Y_{ij} = f\left(\Phi_i, X_{ij}\right) + \varepsilon_{ij} \qquad \text{(Eq.7)}$$

Here, Y_{ij} denotes the value of the dependent variable measured in the jth measurement on the ith sample tree, X_{ij} the value of the independent variable measured in the jth measurement on the ith sample tree, Φi the parameter values of the model, and $\mathcal{E}_{ij}$ the model errors (Castedo-Dorado et al., 2006; Crecente-Campo et al., 2010). The division of model parameters by mixed models into two groups, namely, fixed-effect and random-effect parameters can be demonstrated with the following formula (*Eq. 8;* Castedo-Dorado et al., 2006).

$$\Phi_i = A_{ij}\beta + B_{ij}b_i \qquad \text{(Eq.8)}$$

Here, β is the fixed effect parameter and is calculated for the entire population, while b_i is the random-effect parameter, and indicates the variation among the sample trees. In mixed models, the fundamental assumption for the random-effect parameter and model errors is as follows (*Eqs. 9* and *10*):

$$b_i \sim N(0, D) \qquad \text{(Eq.9)}$$

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$$\varepsilon_{ij} \sim N(0, R) \quad \text{(Eq.10)}$$

Here, b_i (random-effect parameter) is normally distributed with an arithmetic mean of 0 and a variance of D, and $\mathcal{E}_{ij}$ (model error) is also normally distributed with an arithmetic mean of 0 and a variance of R. The estimation of the components in these assumptions constitutes a significant aspect of mixed models (Lappi, 1997). While component D is the positive-definite variance-covariance matrix that represents the variation among the sample trees (variation among trees), component R is the variance-covariance matrix that defines the variation among the data obtained from the sample trees (variation within a tree). The formulas of such variance-covariance matrices that define and model the variation both among sample trees and among the data measured on sample trees are shown below (*Eqs. 11* and *12*):

$$D = \begin{bmatrix} \sigma_u^2 & \sigma_{uv}^2 \\ \sigma_{uv}^2 & \sigma_v^2 \end{bmatrix} \quad \text{(Eq.11)}$$

$$R = \sigma^2 I_i \quad \text{(Eq.12)}$$

Here, σ_u^2 denotes the variance of random-effect parameter u, σ_v^2 the variance of random-effect parameter v, σ_{uv}^2 the covariance between the random-effect parameters, σ^2 the value of the model error, and I_i the value of the diagonal matrix defining the non-constant variance, in which the number of rows and columns is equal to the number of items of data to be used for the sample tree on which the mixed-effects model will be applied (Castedo-Dorado et al., 2006; Trincado et al., 2007).

The variance components and fixed parameters of the best compatible nonlinear stem taper model were estimated with PROC NLMIXED procedures of the SAS/ETS 9.3 package (SAS Institute, 2013). Using this method, different random parameter combinations for the best stem taper model were tested. As the criteria for determining the best random-effect parameter combination, Akaike's Information Criterion (AIC), Schwarz's Bayesian Information Criterion (BIC) and twice the negative log-likelihood (-2Ln(L)) were used.

Calibration response

Another important issue that needs to be considered and resolved in mixed-effects modeling is the calibration responses of the model. Calibrated models allow more accurate, consistent and reliable estimations to be made (Castedo-Dorado et al., 2006; Trincado and Burkhart, 2006; Yang et al., 2009; Crecente-Campo et al., 2010; Cao and Wang, 2011). In mixed-effects models, the observed values obtained newly from the sample areas are used to calculate random parameters, and by adding these random parameter values (or subtracting, if negative) to the fixed effect parameter values that apply to the entire population, the parameter values that apply to the sample area in question are calculated. In calibrating mixed models in forestry applications, the Best Linear Unbiased Predictor (BLUPs) method employed first by Lappi (1997) in a forestry study, and also known as Henderson's equations, is used.

The Best Linear Unbiased Predictor (BLUPs) method requires the measurement of a certain amount of new data in the sample area of the growth environment to be calibrated, particularly for the estimation of the random-effect parameter (Crecente-

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Campo et al., 2010). In particular, the determination of which trees (the thickest, thinnest or medium-diameter trees) will be measured in the sample area is referred to as the calibration response of mixed-effects models. For this purpose, random parameters are calculated using a varying number of trees with different qualities, and subsequently, the error values of the estimations made are analyzed. Using this method, the random-effect parameter is estimated by using *Equation 13*.

$$\widehat{b}_i \approx DZ_i'\left(R + Z_i DZ_i'\right)^{-1}\left(Y_i - A_{ij}\beta\right) \quad \text{(Eq.13)}$$

Here, components D and R denote the pre-defined variance-covariance matrices, component Z_i the design matrix for random-effect parameters, and Z_i' the inverse of the matrix Z_i. Furthermore, component (Y_i-$A_{ij}\beta$) in the equation above is calculated by subtracting the estimation made using only the fixed effect parameters in the mixed model from the observed value.

To determine the calibration response of the mixed-effects models, the Root-Mean-Squared Error (RMSE), Sum of Squared Error (SSE) and Mean Squared Error (MSE) values were used to identify the option that gives the best result among the compared options (Castedo-Dorado et al., 2006).

Results and discussion

Stem taper model selection

The values for the various model success criteria concerning the stem taper models for both species in this study are given in *Table 3*. As can be seen in *Table 2*, the evaluation of the values concerning the model success criteria identified Model 5, developed by Jiang et al. (2005), as the most successful model for both tree species, explaining 98.7% of the variance in the stem taper estimation for fir trees and 98.3% for spruce trees. The RMSE, $\bar{E}$, MAE, AIC and BIC values of the most successful stem taper equation for firs and spruces were also found to be 1.700 and 1.814 cm, 0.143 and 0.167 cm, and 1.179 and 1.269 cm, 4142.2 and 3951.7, 4148.5 and 3957.8 respectively (*Table 3*).

The equation of Jiang et al. (2005) has produced relatively successful results in the modeling of stem tapers in numerous studies. Using this four-segmented polynomial equation structure (Jiang et al., 2015) to model the stem taper, the adjusted coefficient of determination (R^2_{adj}) and Root-Mean-Square Error (RMSE) values were found to be 98.37% and 1.2798 cm in Jiang et al.'s (2005) study, and 98.4% and 0.7505 for Balsam fir, 98.2% and 0.8517 for Red spruce, and 97.2% and 1.4205 cm for White pine in the study conducted by Li and Weiskittel (2010), 98.13% and 1.3848 cm in the study by Özçelik and Bal (2013), 94.44% and 2.2029 cm in the study by Atalay (2014), 98.43% and 0.9843 cm in the study by Kurt (2014), 97.7% and 1.6302 cm in the study by Kumaş and Kahriman (2015), 92.6% and 2.4190 cm in the study by Doyog et al. (2017), and 97.6% and 1.4755 cm in the study by Şenyurt et al. (2017), respectively.

The parameter estimates, standard error values, t-test values and significance levels within the most successful stem taper model (M5: Jiang et al., 2005) for both species are given in *Table 4*.

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Table 2. *Stem taper functions evaluated based on fitting data*

Model	Expression
M1: Demaerschalk (1972)	$d = b_1 D^{b_2}(H-h)^{b_3} H^{b_4}$
M2: Demaerschalk (1973)	$d = \{b_1 D^2[(H-h)^{b_2}/(b_3 H^{b_2+1} + b_4 H^{b_2})]\}^{0.5}$
M3: Max and Burkhart (1976)	$d = D[b_1(Z-1) + b_2((Z)^2-1) + b_3(a_1 - Z)^2 I_1 + b_4(a_2 - Z)^2 I_2]^{0.5}$ $I_1 = 1, if\ \ Z \leq\ a_1; 0\ otherwise$ $\qquad I_2 = 1, if\ \ Z \leq\ a_2; 0\ otherwise$ $\qquad Z = \frac{h}{H}$
M4: Parresol et al. (1987)	$d = D\{Z_i^2(b_1 - b_2 z_1) + (z_i - a)^2[b_3 + b_4(z_i + 2a)]I\}^{0.5}$ $I = 1, if\ \ Z \geq\ a; 0\ otherwise$ $\qquad Z = \frac{H-h}{H}$
M5: Jiang et al. (2005)	$d = \left\{ \begin{array}{l} I_S\left[D^2\left(1 + \dfrac{\left(1-\frac{h}{H}\right)^{b_1} - \left(1-\frac{1,30}{H}\right)^{b_1}}{1-\left(1-\frac{1,30}{H}\right)^{b_1}}\right)\right] \\ +I_B\left[D^2 - \dfrac{(D^2-F^2)\left(\left(1-\frac{1,30}{H}\right)^{b_2} - \left(1-\frac{h}{H}\right)^{b_2}\right)}{\left(1-\frac{1,30}{H}\right)^{b_2} - \left(1-\frac{5,30}{H}\right)^{b_2}}\right] \\ +I_T\left[F^2\left(\left(\frac{h-5,30}{H-5,30} - 1\right)^2 + I_M\left(\frac{1-b_4}{b_3^2}\right)\left(b_3 - \frac{h-5,30}{H-5,30}\right)^2\right)\right] \end{array} \right\}^{0,5}$ $I_S = 1, if\ \ h <\ 1.3; 0\ otherwise$ $\qquad I_B = 1, if\ \ 1.3 \leq h \leq\ 5.3; 0\ otherwise$ $I_T = 1, if\ \ h >\ 5.3; 0\ otherwise$ $\ I_M = 1, if\ \ h <\ (5,30 + b_3(H-5,30)); 0\ otherwise$
M6: Fang et al. (2000)	$d = c_1\left[H^{(k-b_1)/b_1}(1-Z)^{(k-\beta)/\beta}\alpha_1^{I_1+I_2}\alpha_2^{I_2}\right]^{0.5}$ $Z = \frac{h}{H}$ $\qquad p_1 = \frac{h_1}{H}$ $\qquad p_2 = \frac{h_2}{H}$ $\qquad I_1 = 1, if\ \ p_1 \leq q \leq\ p_2; 0\ otherwise$ $I_2 = 1, if\ \ p_2 \leq q \leq\ 1; 0\ otherwise$ $\beta = b_1^{1-(I_1+I_2)} b_2^{I_1} b_3^{I_2}$ $\qquad c_1 = \sqrt{\frac{a_1 D^{a_2} H^{a_3 - k/b_1}}{b_1(r_0 - r_1) + b_2(r_1 - \alpha_1 r_2) + b_3(\alpha_1 r_2)}}$ $\alpha_1 = (1-p_1)^{(b_2-b_1)k/b_1 b_2}$ $\qquad \alpha_2 = (1-p_2)^{(b_3-b_2)k/b_2 b_3}$ $r_0 = ((1-h_{st})/H)^{k/b_1}$ $\qquad r_1 = (1-p_1)^{k/b_1}$ $\qquad r_2 = (1-p_2)^{k/b_2}$

d is diameter outside bark at a height *h* (cm), *D* is diameter outside bark at breast height (cm), *h* is height above ground to the measurement point (m), *H* is total tree height (m), a_1 and a_2 are the join points to be estimated from the sample data for Max and Burkhart (1976) model, *a* is the join point to be estimated from the sample data for Parresol et al. (1987) model, *F* diameter outside bark at 5.3 m above ground (cm), h_1 and h_2 are the heights from ground level where the two inflection points assumed in the Fang et al. (2000) model occur, *k* is equal to π/40 000, a metric constant for converting from diameter squared in square centimeters to cross-sectional area in square meters, h_{st} is stump height (m), a_1-a_3, b_1-b_4, p_1-p_2 are coefficients to be estimated

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Table 3. *Goodness of fit statistics of six taper model for Caucasian fir and Oriental spruce species*

Taper model	Caucasian fir						Oriental spruce					
	R^2_{adj}	RMSE	$\bar{E}$	MAE	AIC	BIC	R^2_{adj}	RMSE	$\bar{E}$	MAE	AIC	BIC
M1	0.979	2.134	0.006	1.58	4503.5	4509.8	0.977	2.082	0.057	1.57	4155.4	4161.6
M2	0.977	2.245	0.192	1.65	4584.6	4590.9	0.975	2.182	0.203	1.63	4224.8	4231.0
M3	0.978	2.162	0.265	1.59	4527.8	4536.6	0.976	2.147	0.311	1.65	4203.8	4212.4
M4	0.972	2.464	0.577	1.84	4728.2	4735.7	0.967	2.503	0.676	1.96	4429.3	4436.7
M5	0.987	1.700	0.143	1.18	4142.2	4148.5	0.983	1.814	0.167	1.27	3951.7	3957.8
M6	0.981	2.035	0.054	1.50	4434.8	4446.2	0.979	2.001	0.138	1.52	4101.3	4112.4

Table 4. *The parameter estimations for the Jiang et al. (2005) taper model*

Parameters	Caucasian fir				Oriental spruce			
	Estimate	Standard error	t value	p	Estimate	Standard error	t value	p
b_1	158.2541	5.8746	26.94	<0.0001	152.8303	6.5333	23.39	<0.0001
b_2	1.3388	0.5260	2.55	0.0110	2.9909	0.7597	3.94	<0.0001
b_3	0.8796	0.0050	177.66	<0.0001	0.9003	0.0054	168.08	<0.0001
b_4	4.0340	0.1203	33.53	<0.0001	4.0662	0.1576	25.80	<0.0001

In the equation of Jiang et al. (2005), it is hard to measure the diameter at a height of 5.30, although this variable can be estimated using the tree's height and diameter at breast height. For both tree species, the equation for estimating the diameter value at a height of 5.30 m using the height and diameter at breast height values is given below (*Eqs. 14* and *15*).

for Caucasian fir
$$F_{d5.3} = D\left(0.923 + \left(-0.832\frac{5.27}{H}\right)^2\right) \qquad \text{(Eq.14)}$$

for Oriental spruce
$$F_{d5.3} = D\left(0.893 + \left(-0.668\frac{5.27}{H}\right)^2\right) \qquad \text{(Eq.15)}$$

where $F_{d5.3}$ is diameter at a height of 5.30 m, *D* is diameter at breast height (cm), and *H* is total tree height (m).

All parameters in the equations concerning Caucasian fir and Oriental spruce were found to be statistically significant at a significance level of $p < 0.001$. The adjusted coefficients of determination (R^2_{adj}), Root-Mean-Square Error (RMSE), mean errors ($\bar{E}$) and mean absolute errors (MAE)) for both species were found to be 0.986 and 0.987, 1.725 and 1.520 cm, 0.044 and 0.015 cm, and 1.279 and 1.145 cm, respectively.

Nonlinear mixed-effect modeling

In this study, the parameters of the Jiang et al. (2005) equation, as the most successful equation in modeling the diameter of trees along their stem, were also

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estimated through mixed-effects modeling. A test was made of the one, two, three and four-way random-effect parameter combinations of parameters b_1, b_2, b_3 and b_4 of the stem taper model, thus testing a total of 15 different combinations. The Akaike's Information Criterion (AIC), Schwarz's Bayesian Information Criterion (BIC) and twice the negative log-likelihood (-2LnL (L)) were used to determine the best random-effect parameter combination, and the garnered data is presented in *Table 5*.

Table 5. *The goodness-of-fit statistics for different random parameters for the Jiang et al. (2005) taper model*

Random parameters	Caucasian fir				Oriental spruce			
	RMSE	**-2LnL**	**AIC**	**BIC**	**RMSE**	**-2LnL**	**AIC**	**BIC**
b_1	1.657	4090.7	4104.7	4113.5	1.770	3904.4	3918.4	3927.0
b_2	1.706	4137.3	4151.3	4160.1	1.828	3944.8	3958.8	3967.4
b_3	1.239	3629.0	3643.0	3651.8	1.223	3357.4	3371.4	3380.0
b_4	1.196	3572.5	3586.5	3595.3	1.215	3348.2	3362.2	3370.8
b_1,b_2	1.716	4145.2	4163.2	4174.6	1.785	3908.9	3926.9	3937.9
b_1,b_3	1.166	3530.8	3548.8	3560.1	1.144	3258.1	3276.1	3287.1
b_1,b_4	1.119	3465.7	3483.7	3495.1	1.136	3247.3	3265.3	3276.4
b_2,b_3	1.250	3641.6	3659.6	3671.0	1.215	3346.8	3364.8	3375.8
b_2,b_4	1.210	3590.4	3608.4	3619.7	1.208	3338.1	3356.1	3367.2
b_3,b_4	1.100	3439.1	3457.1	3468.5	1.129	3229.1	3247.1	3258.2
b_1,b_2,b_3	1.161	3525.6	3539.6	3548.4	1.125	3233.6	3247.6	3256.2
b_1,b_2,b_4	1.113	3458.6	3472.6	3481.4	1.117	3223.6	3237.6	3246.2
b_1,b_3,b_4	2.817	4918.3	4932.3	4941.1	2.862	3920.8	3934.8	3943.0
b_2,b_3,b_4	2.893	4625.4	4639.4	4648.0	2.983	3426.1	3440.1	3447.8
b_1,b_2,b_3,b_4	2.917	3525.1	3539.1	3546.4	2.302	4191.9	4205.9	4214.4

AIC, BIC and -2LnL values are preferred over coefficients of determination when comparing mixed-effects nonlinear regression models, with regression models that produce smaller information criteria values being considered successful (Castedo-Dorado et al., 2006). As can be seen in *Table 5*, the values of AIC, BIC and -2LnL indicate that the model with random-effect parameters b_3 and b_4 is the most successful for both fir and spruce. Including more than two random parameters either produced failed model convergence or non-significant parameters at a significance level of 0.05 for some variance and covariance parameters and fixed parameters for both species. The parameter estimations of the mixed-effects model are given in *Table 6*.

Also compared nonlinear and mixed effects models to Jiang et al. (2005) stem taper model. These results indicated that all mixed effects models except for the model with random-effect parameters (b_2), (b_1,b_2), (b_1,b_3,b_4), (b_2,b_3,b_4), (b_1,b_2,b_3,b_4) for fir and (b_2), (b_1,b_3,b_4), (b_2,b_3,b_4), (b_1,b_2,b_3,b_4) for spruce provided much better fits than their fixed model counterparts (*Tables 3* and *5*). The reduction rates in AIC and BIC for fir and spruce were 12.8% (1.0-16.5), 13.6% (1.0-18.1) and 12.7% (1.0-16.4%), 13.4% (1.0-18.0) respectively. Mixed effects models were more accurate and precise than those fitted without random effects as root-mean-square error (RMSE) was reduced by 28.2% (2.5-35.5) for fir and 29.5% (2.4-38.4%) for spruce growth prediction. The reduction rates in RMSE, AIC and BIC for fir and spruce were 35.3, 16.2, 16.1% for fir and 37.8, 17.8, 17.7% for spruce with random-effect parameters b_3 and b_4.

***Table 6.** The parameter estimations and variance components with the goodness-of-fit statistics for the selected stem taper model (M5: Jiang et al., 2005)*

Parameters		Caucasian fir				Oriental spruce			
		Estimate	**Standard error**	**t value**	**p**	**Estimate**	**Standard error**	**t value**	**p**
Fixed parameters	b_1	158.26	3.5914	44.07	<0.001	152.90	4.0306	37.93	<0.001
	b_2	1.3388	0.3007	4.95	0.0110	2.9898	0.4588	6.52	<0.001
	b_3	0.8522	0.0034	251.59	<0.001	0.8473	0.0040	211.49	<0.001
	b_4	3.6780	0.1271	28.94	<0.001	3.4051	0.1381	24.65	<0.001
Variance component	σ_u^2	0.0107	0.0014	7.69	<0.001	0.0132	0.0024	5.51	<0.001
	σ_v^2	4.2857	0.7841	5.47	<0.001	3.3346	0.6879	4.85	<0.001
Covariance	σ_{uv}^2	0.1879	0.0296	6.35	<0.001	0.1723	0.0359	4.80	<0.001
Error	σ^2	0.9470	0.0327	28.93	<0.001	1.0648	0.0384	27.74	<0.001

The relative height-error distribution in the nonlinear and mixed-effects models for Caucasian fir and Oriental spruce is given in *Figures 4* and *5*, respectively. For both species, the model with random-effect parameters has a more homogenous error variance structure than that of the nonlinear model at all relative height values (*Figs. 4* and *5*). In other words, as is seen in *Figures 4* and *5,* the error variance values rise in parallel to the relative height values in the nonlinear model but remain constant in the mixed-effects model (Trincado and Burkhart, 2006; Yang et al., 2009; Sharma and Parton, 2009; Özçelik et al., 2011; Özçelik and Yaşar, 2015; Şenyurt et al., 2017).

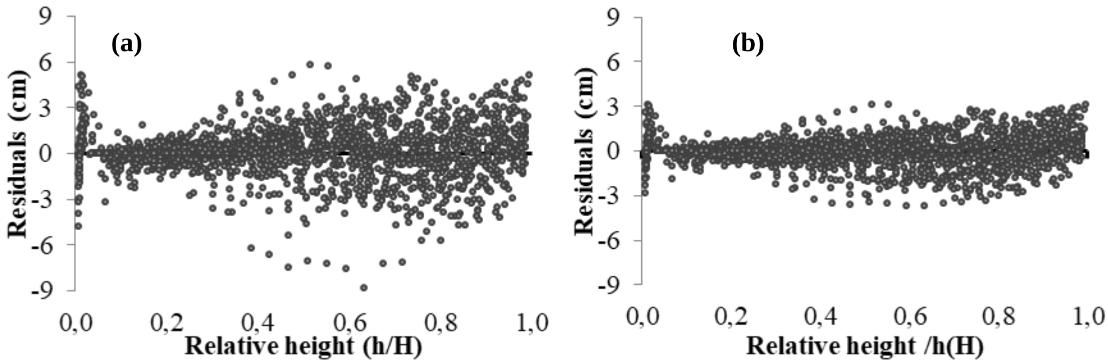

***Figure 4.** Residual plots for the (a) nonlinear and (b) mixed effects model for Caucasian fir*

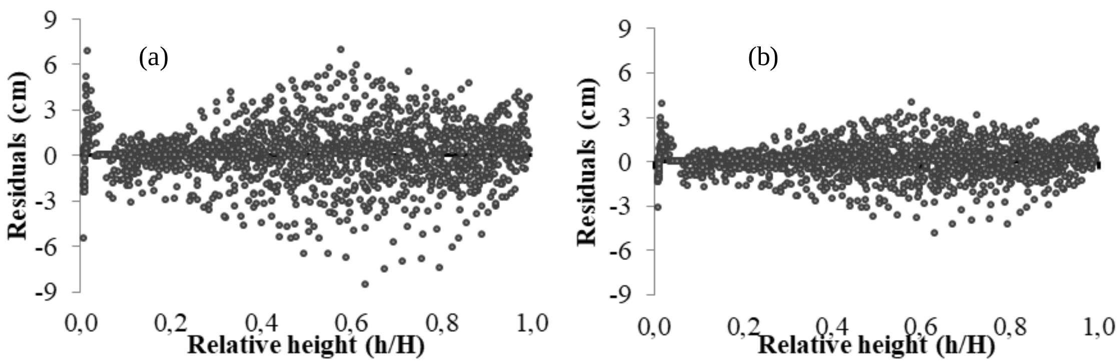

***Figure 5.** Residual plots for the (a) nonlinear and (b) mixed effects model for Oriental spruce*

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The error autocorrelations for Lag 1 for Caucasian fir and Oriental spruce are given in *Figures 6* and *7*, respectively. Here, a positive correlation is observed in the nonlinear regression model, and the error-related autocorrelation problem is remedied once the random-effect parameters are added to the model for both species. (*Figs. 6b* and *7b*).

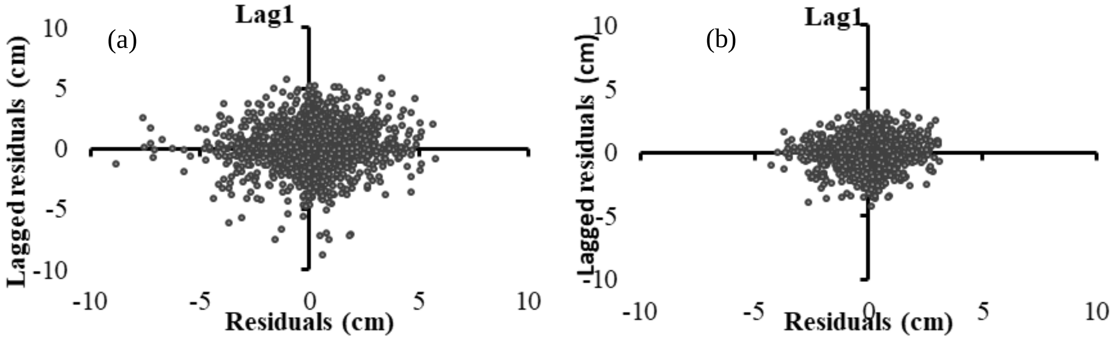

***Figure 6.** Residuals plotted against lagged residuals for (a) nonlinear regression Jiang et al. (2005) model and (b) nonlinear mixed effect regression model of Caucasian fir*

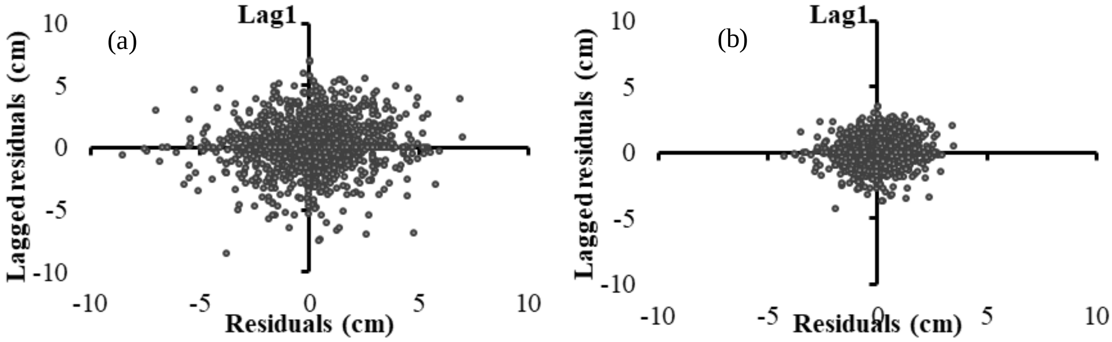

***Figure 7.** Residuals plotted against lagged residuals for (a) nonlinear regression Jiang et al. (2005) model and (b) nonlinear mixed effect regression model of Oriental spruce*

As obtained in some studies (Trincado and Burkhart, 2006; Sharma and Parton, 2009; Gomez-Garcia et al., 2013; Özçelik and Yaşar, 2015; Ercanlı, 2015; Şenyurt et al., 2017; Zheng et al., 2017), the error (RMSE) of the models decreased from 1.700 to 1.100 cm for Caucasian fir and from 1.814 to 1.129 cm for Oriental spruce in the transformation of the nonlinear model to the mixed-effects model (*Tables 3* and *5*).

For the compatible stem taper model of Jiang et al. (2005) to set an example for taper estimations through the use of nonlinear mixed-effects regression parameters, the graphical results of the taper estimations for three trees in the small-, medium- and large-diameter classes are given in *Figure 8*. The trees were randomly selected from approximately thin, medium and thick diameters at breast height. The diameters of these trees being 22.4, 38.4 and 60.1 cm for firs and 18.2, 36.1 and 56.1 cm for spruces and the heights are 16.2, 26.8 and 36.1 m for firs and 13.9, 23.0 and 31.2 m for spruces. As seen in *Figure 8*, the mixed-effects model is successful to define the stem form of both Caucasian fir and Oriental spruce trees, and similar results have been obtained in previous studies in which the stem taper model was obtained through mixed-effects modeling (Trincado and Burkhart, 2006; Yang et al., 2009; Sharma and Parton, 2009; Li

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and Weiskittel, 2010; Subedi and Sharma, 2011; Gomez-Garcia et al., 2013; Chiu et al., 2015; Özçelik and Yaşar, 2015; Şenyurt et al., 2017).

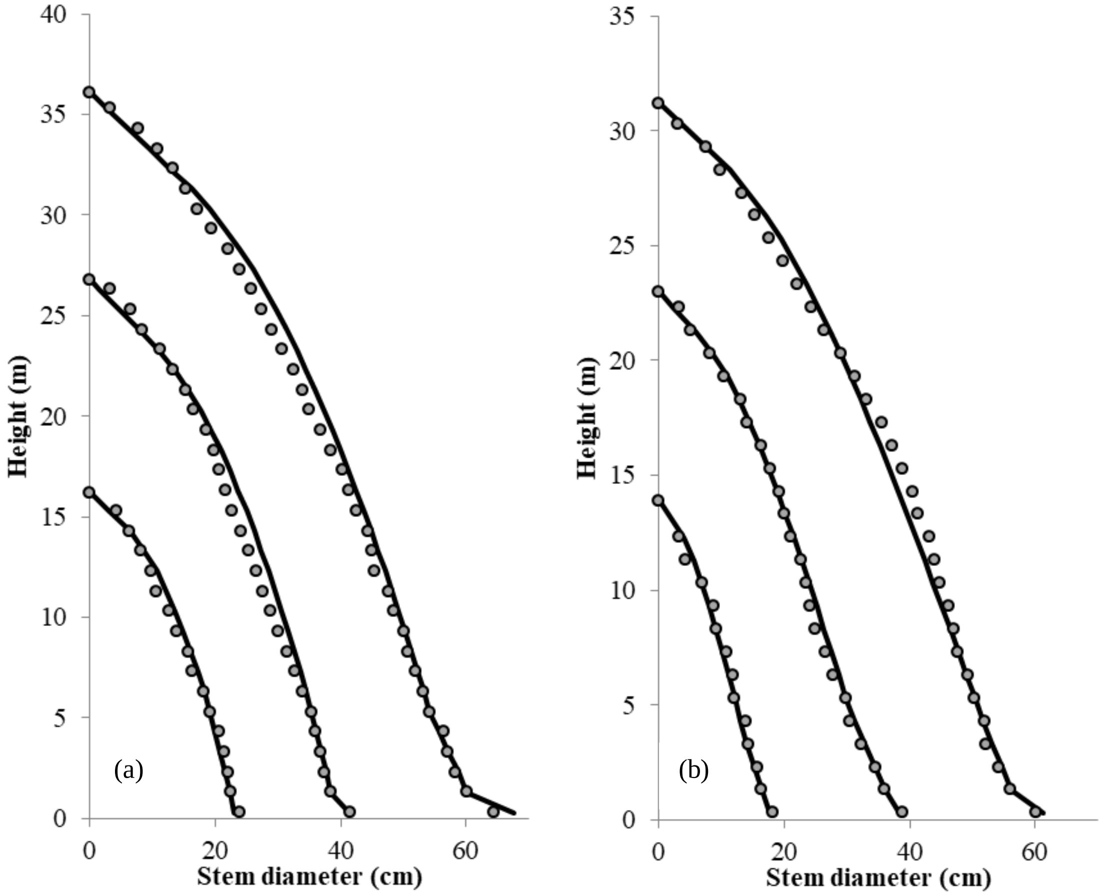

Figure 8. *Stem profile curves of taper equations to three sampled trees using mixed-effect model with b3 and b4 having random effects for (a) Caucasian fir and (b) Oriental spruce*

One of the major steps in the mixed-effects modeling process is the presentation of the calibration responses of the model that is found to be the most successful in modeling the stem taper. For this purpose, the model of Jiang et al. (2005) was calibrated using data related to 22 samples trees from each species that had not been used in the formation of the models or the estimation of parameters. In determining the calibration responses of mixed-effects models, the different scenarios suggested by Garber and Maguire (2003), Trincado and Burkhart (2006), Yang et al. (2009), Sharma and Parton (2009), Özçelik et al. (2011), Cao and Wang (2011), Subedi and Sharma (2011), Gómez-García et al. (2013, 2016), and Şenyurt et al. (2017) were also considered. These calibration response options are as follows: (i, ii, iii) 3, 4 or 5 diameter values at the lowest base section of trees; (iv, v, vi) 3, 4 or 5 diameter values obtained by dividing the measured diameters into two equal parts of sample trees; (vii, viii, ix) 3, 4, or 5 diameter values from the top sections of the trees; (x) a total of 3 diameter values from heights of 0.30 m, 1.30 m and 5.30 m; (xi) a total of 4 diameter values at heights of 0.30 m, 1.30 m, 5.30 m and in the top section of trees; (xii) a total of 5 diameter values from heights of 0.30 m, 1.30 m and 5.30 m, the diameter that divides

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the measured diameter values into two equal parts of tree ($d_{h/2}$) and the diameter of the top section.

With *Equation 13* regarding the Best Linear Unbiased Predictor (BLUP) method, the random-effect parameters for the sample trees were calculated using the SAS program code of Trincado et al. (2007). The random-effect parameter values (parameters u and v) that were calculated in 12 different ways for each sample tree from both species (22 sample trees) were added to parameters b_3 and b_4. As a result, when the values of parameters b_3 and b_4 were changed for each sample tree, different stem taper equations were obtained for these trees. Subsequently, in the calibration of mixed-effects models, the estimations for the diameter values measured along the stems of 22 sample trees were obtained using different stem taper equations, obtained through random parameters, and calculated based on 12 different calibration options. The Sum of the Squared Error (SSE), Mean Squared Error (MSE) and Root-Mean-Squared Error (RMSE) values for the estimations related to the sample trees from each species are given in *Table 7*. The best estimation results from among these calibration options were obtained from the calibration option based on the measurement of five diameters from the middle section of firs (SSE: 451.0, MSE: 0.9999 and RMSE: 1.0090) and four diameters from the stump section of spruces (SSE: 548.0, MSE: 1.1760 and RMSE: 1.0939).

Table 7. *Comparison of the predictive performance for some sampling scenarios*

Calicration scenarios	**Caucasian fir**			**Oriental spruce**		
	SSE	**MSE**	**RMSE**	**SSE**	**MSE**	**RMSE**
(i) Three diameter-base of the stem	479.2	1.0626	1.0401	580.1	1.2448	1.1254
(ii) Four diameter-base of the stem	460.3	1.0205	1.0193	548.0	1.1760	1.0939
(iii) Five diameter-base of the stem	460.3	1.0206	1.0193	580.5	1.2457	1.1258
(iv) Three diameter-middle part of stem	451.2	1.0005	1.0093	566.5	1.2157	1.1122
(v) Four diameter-middle part of stem	451.3	1.0008	1.0094	567.7	1.2183	1.1134
(vi) Five diameter-middle part of stem	451.0	0.9999	1.0090	567.1	1.2169	1.1127
(vii) Three diameter-top of stem	458.4	1.0165	1.0173	579.0	1.2426	1.1244
(viii) Four diameter-top of stem	457.4	1.0143	1.0162	577.8	1.2400	1.1232
(ix) Five diameter-top of stem	456.6	1.0124	1.0152	576.7	1.2376	1.1222
(x) Three diameter-$d_{0.30}$, $d_{1.30}$, $d_{5.30}$	491.2	1.0891	1.0530	575.6	1.2353	1.1211
(xi) Four diameter-$d_{0.30}$, $d_{1.30}$, $d_{5.30}$, d_{top}	459.8	1.0196	1.0188	577.0	1.2383	1.1225
(xii) Five diameter-$d_{0.30}$, $d_{1.30}$, $d_{5.30}$,$d_{h/2}$,d_{top}	458.2	1.0160	1.0170	575.3	1.2345	1.1207

Conclusions

In this study, a compatible stem taper model is developed that uses the mixed-effects modeling technique for the mixed stands of Caucasian fir and Oriental spruce in the Artvin-Ardanuç region. To model the stem taper of the trees, six compatible stem taper models developed by Demaerschalk (1972, 1973), Max and Burkhart (1976), Parresol et al. (1987), Fang et al. (2000) and Jiang et al. (2005) were used.

An evaluation of the success criteria values revealed that the model developed by Jiang et al. (2005) was the most accurate in modeling the stem diameter for both species (*Table 3*). The model developed by Jiang et al. (2005) used in this study explains 98.7% of the variance in the stem taper estimation for fir and 98.3% of the variance in the stem

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taper estimation for spruce (*Table 3*). The RMSE values in the most successful stem taper equation for firs and spruces were found to be 1.700 and 1.814 cm, respectively.

The most significant advantage of the mixed-effects modeling technique over conventional regression models is that mixed-effects modeling technique allows the elimination of the problem of autocorrelation among the data and the homogenization of the distribution of error variance. In this study, the autocorrelation problem was all but eliminated using the mixed-effects modeling technique, and the distribution of error variance was transformed into a homogenous structure for all relative height classes. After the addition of the random-effect parameters to the model for both species, the error-related autocorrelation problem was eliminated (*Figs. 6b* and *7b*), and the model with random-effect parameters had a more homogenous error variance structure at all relative height values (*Figs. 4b* and *5b*). Moreover, the decrease in the RMSE values in the mixed-effects modeling approach from 1.700 cm to 1.100 cm for Caucasian fir and 1.814 cm and 1.129 cm for Oriental spruce in the transformation from the nonlinear regression model is another successful aspect. Mixed effects models provided much better fits than their fixed model counterparts for both species.

Calibrated models allow more accurate, consistent and reliable estimations to be made (Trincado and Burkhart, 2006; Yang et al., 2009; Cao and Wang, 2011). The best estimation results from the 12 different calibration options in this study were obtained through the calibration option, which involved the measurement of five diameters from the middle section of firs and four diameters from the stump section of spruces. In studies by different researchers, calibration responses are analyzed using the values of one, two, three, four, or five diameters at various stem heights (Trincado and Burkhart, 2006; Yang et al., 2009; Lejeune et al., 2009; Sharma and Parton, 2009; Özçelik et al., 2011; Gómez-García et al., 2013; Şenyurt et al., 2017).

All of these assessments have shown that successful diameter estimations could be made using the nonlinear mixed-effects modeling technique for the stem diameter estimations of both species in mixed stands of Caucasian fir and Oriental spruce in the Artvin-Ardanuç region. Thus, the success of the diameter estimations has a direct effect on volume estimations. In practical forestry applications, it is more important to accurately estimate the volume of trees than their diameter at breast height diameter. The mixed-effects model approach is substantial, and will allow forest managers to better predict tree volumes to any utilization standard. It is worth noting that when one of the models with a similar level of success in estimation needs to be selected for a specific species in any region, the practical implementation of the method and the preferences of the forest managers is also important.

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